Kinematics Guide

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Learning when and how to use the kinematic equations can be difficult starting out, so it's helpful to have a little guide to act as a crutch in the beginning. This guide is not to be used as a substitute for studying, but merely as a supplement until you have gained enough intuition that you no longer need this guide.

First, we write down the three basic kinematic equations of Newtonian mechanics. Without loss of generality, we shall express the equations in the y-direction. Note that in place of writing the y component of the acceleration, we have written $-g$ instead. This is to emphasize that g has the numerical value of $9.8\,\mathrm{m/s^2}$ as opposed to writing $g = -9.8\,\mathrm{m/s^2}$. The three equations (in the y-direction) are

$$\Delta y = v_{y_0} t - \frac{1}{2} g t^2, \qquad v_{y_f} = v_{y_0} - gt, \qquad v_{y_f}^2 - v_{y_0}^2 = -2g\Delta y, \tag{1}$$

where we have defined $\Delta y \equiv y - y_0$. Determining which equation to use depends heavily on the context. That being said, you want to pick the equation that most encompasses everything you know, while containing as few things as you don't know. This will be true for all topics in this class (as well as your other physics classes). Within the context of the class material, this means you want to use the equation where you know at least two of the variables (not including constants like g) and you are reduced to finding the third. To that end, we present the following table:

Known	Want To Know	Equation
t, v_{y0}	Δy	$\Delta y = v_{y0}t - \frac{1}{2}gt^2$
$v_{y0}, \Delta y$	t	$\Delta y = v_{y0}t - \frac{1}{2}gt^2$
$\Delta y, t$	v_{y0}	$\Delta y = v_{y0}t - \frac{1}{2}gt^2$
v_{y0}, t	v_{yf}	$v_{yf} = v_{y0} - gt$
t, v_{yf}	v_{y0}	$v_{yf} = v_{y0} - gt$
v_{yf}, v_{y0}	t	$v_{yf} = v_{y0} - gt$
$v_{y0}, \Delta y$	v_{yf}	$v_{yf}^2 - v_{y0}^2 = -2g\Delta y$
$\Delta y, v_{yf}$	v_{y0}	$v_{yf}^2 - v_{y0}^2 = -2g\Delta y$
v_{yf}, v_{y0}	Δy	$v_{yf}^2 - v_{y0}^2 = -2g\Delta y$

Lastly, we wish to include a few equations that are also helpful with kinematic problems. Frequently, we will find it necessary to relate the different components of the velocity $\vec{\mathbf{v}}$. Thus, it would be useful to know these quantities relate to one another

$$||\vec{\mathbf{v}}|| \equiv v = \sqrt{v_x^2 + v_y^2}, \qquad \tan\theta = \frac{v_y}{v_x}. \tag{2}$$

Please note we left the components in the more abstract notation v_x and v_y instead of the much more concrete form $v_x = v\cos\theta$ and $v_y = v\sin\theta$. The reason is due to these two relations being *coordinate system dependent* and we prefer to write these equations without making a reference to a particular coordinate system. And remember, v_{x0} will remain *constant* within the context of projectile motion because nothing affects the motion of the projectile in the x-direction when it is in flight.