A Guide for Finding the Electric Field

Marcell Howard

February 14, 2021

Knowing how to find the electric field can be a daunting task when first starting. We are first introduced through the electric field of a point charge

$$\vec{\mathbf{E}} = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}, \tag{1}$$

where r is the distance from the source of the electric field and the point of interest, and $\hat{r}$ is the direction that points from the source to the point of interest. Now a point charge is essentially an object whose charge can be (thought of as being) concentrated (or what physicists call *localized*) into a particular point. The problem comes when we're not dealing with simple point charges, but when the charge is distributed over some space. Fortunately for instances like that, the equation we wrote down previously still holds so long as you concentrate hard enough on a very small patch of the object of interests i.e.

$$\mathrm{d}\vec{\mathbf{E}} = \frac{\mathrm{d}q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}, \tag{2}$$

where $\mathrm{d}q$ is the charge for the tiny patch we are considering and $\mathrm{d}\vec{\mathbf{E}}$ is the tiny electric field is produced by that tiny charge. Now the way we express our tiny charge $\mathrm{d}q$ will depend a lot on how the charges are distributed on the material we're looking at. Assuming a uniform charge density, $\mathrm{d}q$ takes on the following forms:

Charge Type	Charge Density	Volume Element
Line of charge	λ	$\mathrm{d}\ell$
Surface charge	σ	$\mathrm{d}A$
Charge in an object	ρ	$\mathrm{d}V$

where the charge densities λ, σ, and ρ are usually uniform charge densities i.e. charges are spread out equally in the material. Sometimes when finding the electric field, the problem will have certain geometries that we can exploit. When we're in a situation where we have spherical symmetry (inside a ball, or empty space), cylindrical symmetry (inside of a rode, or soda can), or planar symmetry (on top of a sheet, or inside a slab/box) we can use Gauss's law:

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0}, \tag{3}$$

where Q_{enc} is the charge that is enclosed in our Gaussian surface. We have to stress: the area that we're computing the flux through is the area of the Gaussian surface. The Gaussian surface is a fictitious, mathematical object that we construct in order to help us calculate the electric field in some region. Since the only Gaussian surfaces that we construct will be spheres, cylinders, and boxes/pill boxes, we can reduce the electric flux to the following table:

Symmetry	Flux	Direction
Spherical	$4\pi r^2 E$	$\hat{r}$
Cylindrical	$2\pi r L E$	$\hat{r}$
Planar, (both ends in field)	$2AE$	$\hat{n}$
Planar, (one end in field)	AE	$\hat{n}$

where r is the radius of our Gaussian sphere/cylinder, L is the length of the Gaussian cylinder, and A is the area of the Gaussian box. The direction of the electric field in the case of the sphere and cylinder is called *radially outward*, which is to say the direction is perpendicular to the surface of the sphere and cylinder. The direction of the electric field for a planar object will also be perpendicular to the surface as well.